SPROTT STUDENT INVESTMENT FUND

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Low-Volatility Targeting: A Risk-Factor Approach

Abstract

Risk factor styles of investing are widely popular within the active investment management industry due to its ability to generate alpha and reduce risk beyond traditional allocation. Using the Fama and French (2014) five factor model, I assume each factor to be risk-factor style investment securities and construct four minimum volatility portfolios with different techniques of estimating the covariance matrix. Results indicate that low volatility risk factor portfolios produce superior sharpe, less risk and lower likelihood and impact of tail events.

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1 Introduction

In a traditional asset allocation setting, the investor enjoys his free lunch through diversifying his portfolio across a wide range of asset classes and securities. Modern portfolio theory dictates that the investor, under the assumptions of rationality and risk-averse, would allocate cash toward his potential opportunity set in such a way that maximizes his expected utility (Markowitz, 1952). In reality, many practical issues arise when optimizing one's portfolio. Two common issues being estimation error and changes in the nature of diversification. The latter occurs in an environment where portfolio risk increases to an unprecedented level, primarily due to near-unity correlations across all equity classes, and the power of diversification breaks down. See Asness, Israelov and Liew (2011) and Ilmanen and Kizer (2012). The former occurs due to the noisiness of financial data and time-varying parameters. Estimation risk is termed to represent the loss of performance (ex: ex-post utility) as a result of poor estimates of parameters. See Klein and Bawa (1976, section 1). For the rest of this research paper, I will introduce risk-factor style investing in section 2, portfolio construction description in section 3, parameter estimation methods in section 4 and performance results in section 5.

2 Risk-factor Investments

More recent advances from both academics and practitioners have advocated the use of risk factor style of investing in portfolio management. See Ilmanen and Kizer (2012). Risk factors are primarily designed to exploit some form of market inefficiencies that generate abnormal returns. These factors exist with underlying rationale and observed time-persistence. Risk factors also serve as an excellent tool to reduce volatility by having near zero and negative correlations across-factors as well as with traditional asset classes. The exact construction of available risk factors are beyond the scope of this paper[2], however, I will examine a simple set of available data made available by Eugene Fama and Kenneth French (2014). The Fama and French Five factor model is a collection of portfolios designed to proxy returns for firms exhibiting certain financial characteristics ex: the

[2] The BARRA risk-factors are a common set widely found in the investment industry. MSCI has constructed indices for them here.

value anomaly. While most managers use the factor models to help decompose returns for ex-post performance analysis, I will assume them to be investable equity risk-factor style securities[3]. They satisfy the two main properties of producing alpha and having near negative to zero correlations. See below figure.

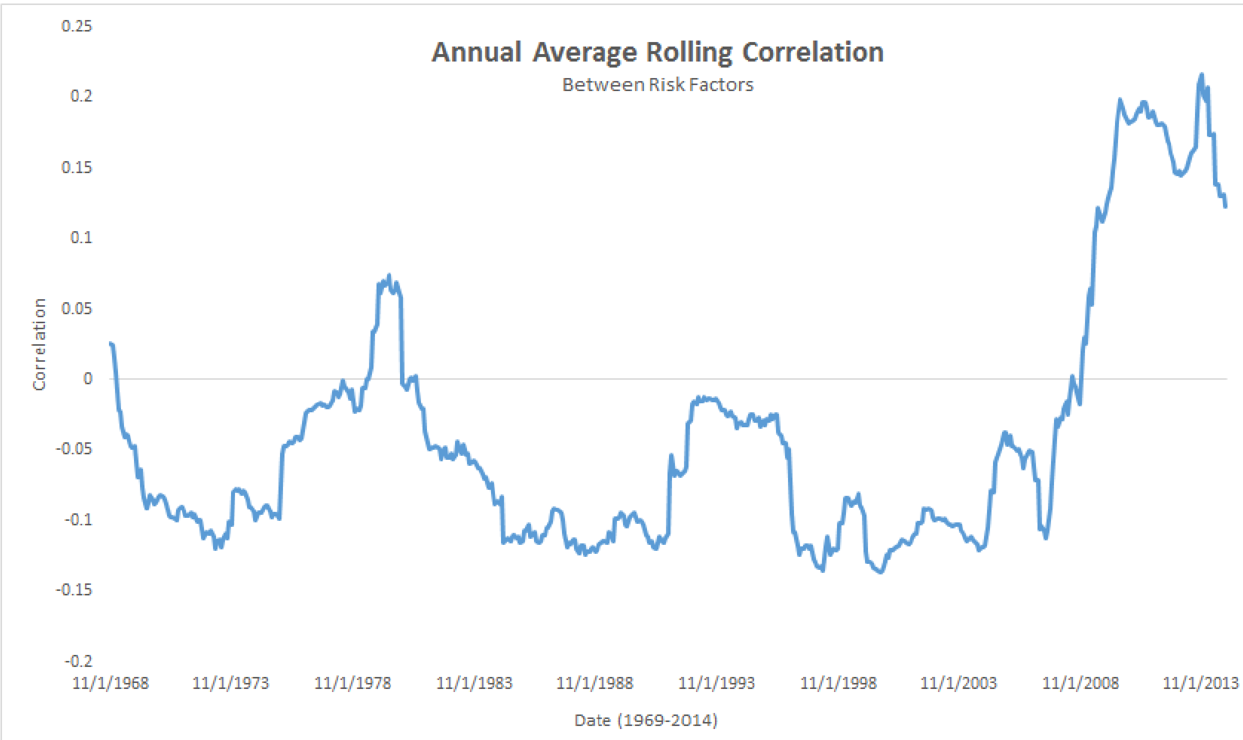

Figure 1 - Average annual rolling correlation between each risk-factor sampled from January 1964 to December 2014. The sample estimate for correlation is from a 5 year window.

The five respective risk factor style securities are (note that all portfolios are value-weighted): *Market*, a portfolio of all securities listed on the CRSP including NYSE, AMEX and NASDAQ; *Size (SMB)*, a portfolio long on small-cap and short on large-cap equities; *Value (HML)*, a portfolio long on high book-to-market and short on low book-to-market; *Profitability (RMW)*, a portfolio long on robust operating profitability and short on weak operating profitability; *Investments (CMA)*, a portfolio long on conservative investments and short on portfolios with aggressive investments.

[3] Risk factor styles can be applied beyond the equity space into other asset classes such as commodities, fixed income, derivatives, credit instruments, etc.

3 Portfolio Construction

Portfolio optimization often requires estimation of expected returns, asset volatility and co-variations between assets. In terms of estimation risk, forecasting expected return is often a difficult task in comparison to forecasting the covariance matrix. As a result, this has given rise to more recent methods of portfolio construction e.g Risk Parity. Having risk factor securities with near zero correlations, we can attempt to construct risk-free portfolios by minimizing the portfolio volatility toward zero. Although in reality, it is impossible to achieve risk-free portfolio, I will show in section 5 that these portfolios can have significantly stable up-trends. The portfolio construction problem can be considered as optimizing weights to minimize variance such that the portfolio is fully invested or more formally

$$\min_{w} w'\Sigma w \quad s.t \quad 1'w = 1$$

Where $w = \{w_1, w_2, ..., w_N\}$ is an $N \times 1$ vector of portfolio percentage weighting and Σ is an $N \times N$ covariance matrix of asset returns. Data from Kenneth French's database offers monthly excess returns from 1964 to 2014 end. Using a five year sampling intervals, I test for out-of-sample performance starting from 1969 to the end of 2014, conditioning parameters only on previously observed data. This is done to prevent any form of in-sample fitting which will overstate performance. Lastly, all portfolios are rebalanced quarterly.

4 Parameter Estimation

Since we are targeting for minimum volatility portfolios, only the covariance matrix will need to be estimated. I present four methods of estimating the covariance matrix: Sample estimate (Naive), Non-parametric bootstrapping [See Singh and Xie (2008) for a summary], Ledoit and Wolf (2003) Shrinkage and Vasicek (1973) Shrinkage methodology. Under the Naive estimation, we just take the sample estimator. Bootstrapping, as introduced by Bradley Efron, is the act of repeated sampling and calculating our estimator of the sampled data. Averaging over all samples, the bootstrap methodology may provide a better estimate of the population covariance matrix. In this study, I will be setting the bootstrap sample size to 5000. Ledoit and Wolf (2003) shrinkage method takes a weighted average of the sample covariance matrix and a prior. I use the prior belief that correlations among each risk factor revert toward zero over time. Calibration of the shrinkage parameter will be estimated the same way it is proposed in the paper through minimizing the frobenius loss[4]. Lastly, the Vasicek (1973) shrinkage was originally proposed as a bayesian estimate between the sample beta and a cross-sectional prior of betas. I will use the same results derived in the analyses and shrink toward the prior belief of average pairwise correlation with a variance of average bootstrapped variance or $\rho_{ij} \sim N(\bar{\rho}, \frac{1}{N}\Sigma\sigma^2{}_{B,ij})$ where $\bar{\rho} = \frac{2}{N(N-1)}\sum_{i=1}^{N}\sum_{j=i+1}^{N}\rho_{ij}$

5 Portfolio Performance

In assessing overall performance, we can benchmark the Market portfolio in comparison to the other portfolios. Table 2 lists all performance metrics from 1969 to 2015. Furthermore, Figure 3 plots a time-series comparison between the three portfolios constructed from different estimation methods.

Metrics	**Market**	**Naive**	**Bootstrap**	**Ledoit**	**Vasicek**
Performance Metrics					
Excess Return	699%	241%	243%	240%	219%
Annual Average	5.814%	2.7%	2.715%	2.697%	2.561%
Annual Volatility	15.91%	2.351%	2.356%	2.356%	2.428%
Sharpe Ratio	0.3654	1.1484	1.1524	1.1447	1.0548
Treynor Ratio	6.99	93.31	96.04	92.05	281.28
Market Risk-Adjusted Performance					
CAPM Alpha	0	0.212%	0.214%	0.212%	0.210%
CAPM Beta	1	0.0258	0.0253	0.0261	0.0078
Risk Properties					

[4] The matlab code can be found here.

95% Value at Risk	-7.57%	-0.88%	-0.87%	-0.89%	-0.93%
Max Drawdown	54.36%	5.92%	5.84%	5.69%	8.87%
Skewness	-0.536	-0.021	-0.015	-0.037	0.008
Excess Kurtosis	1.821	1.484	1.484	1.444	1.153

Table 2 - Portfolio Performance versus the Market Portfolio. Annual Average metric is measured with arithmetic average. Annual Volatility is measured by $\sqrt{12}\sigma$ *where* σ *is equal to the monthly standard deviation*

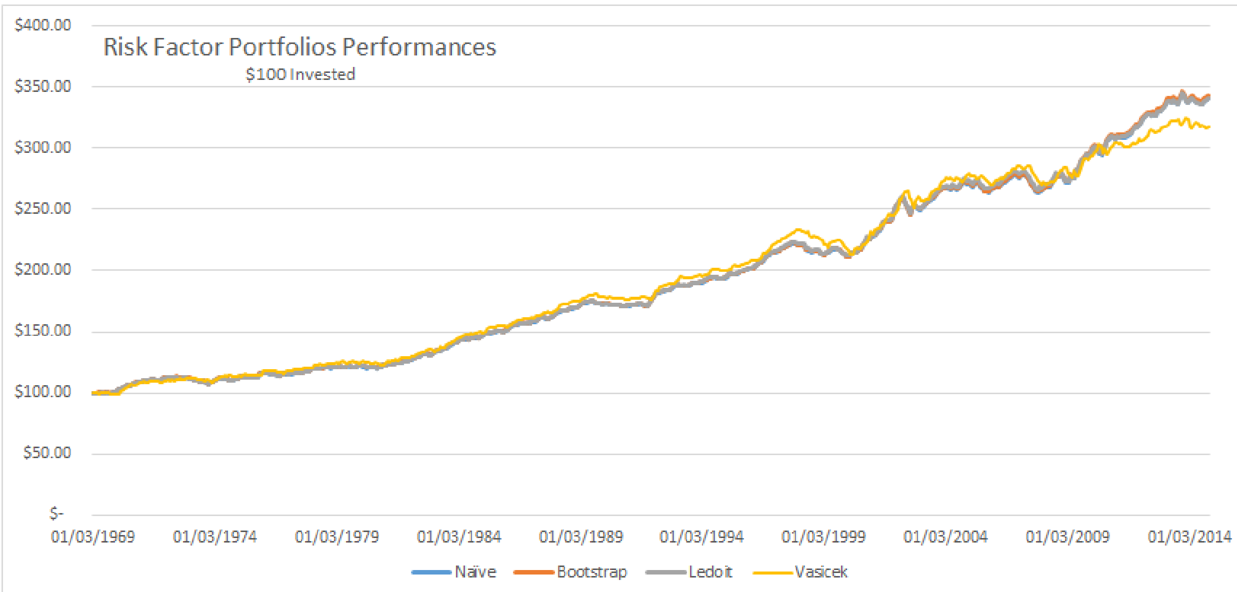

Figure 3 - Portfolios Performance over time. Benchmark is not shown here and only estimation methods are compared.

Overall, the performance characteristics show that all methods were able to outperform the market portfolio on a risk-adjusted basis. Within each estimation method, we find results to be widely similar for sampling, bootstrap and Ledoit shrinkage. Vasicek did underperform in terms of higher volatility and lower return but it can be seen, through the time-series, that there were long intervals where it was netting higher returns than the other methods. Moreover, the method seemed to underperform vastly ever since the 2008 crisis. I hypothesize that this is because the model failed to adjust to rising correlations (see figure 1) among factors and allocations were suboptimal. Lastly, it is peculiar that it had the highest Treynor ratio and lowest likelihood of tail events while maintaining an overall higher volatility. I hypothesize this is primarily due to its low correlation with the market and less idiosyncratic downside volatility. It is also worth noting that the bootstrapping methodology seemed to provide a much better result on all aspects than the sample estimate. We can conclude with some evidence that the non-parametric solution can be used to potentially improve estimation of the covariance matrix. Ledoit shrinkage was similar in nature to sample and bootstrap but underperformed slightly.

6 Conclusion

Through implementing simple risk factor securities and targeting low volatility, I found that they are able to outperform the traditional U.S equity market portfolio on a risk-adjusted basis. Risk measures introduced in table 2 suggest that these portfolios have historically extremely low drawdowns and much less likely to experience tail risk events that equity markets may occasionally go through. While I took a low volatility approach, there are still plenty other viable methods available in the portfolio manager's toolbox for example the popular risk parity approach. Any money manager looking to further diversify their holdings should search for alpha within the risk factor universe. There are many ETFs to service these needs at an affordable cost and weight allocation methods to implement them in a practical manner.

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